A Note on Gauss-Bonnet Holographic Superconductors

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We present an analytic treatment near the phase transition for the critical temperature of (3+1)-dimensional holographic superconductors in Einstein-Gauss-Bonnet gravity with backreaction. We find that the backreaction makes the critical temperature of the superconductor decrease and condensation harder. This is consistent with previous numerical results.

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I. INTRODUCTION

Gauge/Gravity duality stems from string theory provides a rich tool for analyzing strongly coupled field theory [1]. Especially, the duality provides an established method for calculating correlation functions in a strongly interacting field theory using a dual classical gravity description [2, 3]. Holographic superconductors established in [4, 5] are remarkable examples where the Gauge/Gravity duality plays an important role. There, a superconducting phase transition is described by black hole physics according to the duality.

Many of previous works on the holographic superconductors were performed in the probe limit, where the backreaction of matter fields on the spacetime metric is neglected. However, we found that the effect of the Gauss-Bonnet coupling lowers the critical temperature of holographic superconductors in previous work [6]. Backreaction becomes important when considering lower temperature of black holes in AdS spacetime, because lower Hawking temperature of black holes means smaller black holes, i.e., larger Coulomb energy of the matter fields near the black hole horizon.

The critical temperature was obtained numerically both with and without the backreaction [4, 5, 7]. As an analytic approach for deriving the critical temperature, an approximate analytic formula was proposed in the probe limit by a matching method [6]. An alternative analytic method using the expansion around the critical point where the phase transition occurs was also proposed [8]. Some other analytic approaches were also proposed in the probe limit [9–12]. In this note, we derive analytically the critical temperature of the Gauss-Bonnet holographic superconductors with backreaction by combining the small backreaction approximation and the matching method near the critical point.

II. GAUSS-BONNET BLACK HOLES

We begin with the action for a Maxwell field and a charged complex scalar field coupled to the Einstein-Gauss-Bonnet:

$$\begin{aligned} S \;=\; & \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R+\frac{12}{L^2}+\frac{\alpha}{2}\left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}-4R^{\mu\nu}R_{\mu\nu}+R^2\right)\right] \\ & +\int d^5x\sqrt{-g}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-|\nabla\psi-iqA\psi|^2-m^2|\psi|^2\right], \end{aligned} \tag{1}$$

where g is the determinant of a metric $g_{\mu\nu}$ and $R_{\mu\nu\lambda\rho}$, $R_{\mu\nu}$ and R are the Riemann curvature tensor, Ricci tensor, and the Ricci scalar, respectively. q is the charge of the scalar field. We take the Gauss-Bonnet coupling constant α to be positive. Here, the negative cosmological constant term $-6/L^2$ is also introduced. We look for electrically charged plane-symmetric hairy black hole solutions taking the metric ansatz

$$ds^2=-f(r)e^{2\nu(r)}dt^2+\frac{dr^2}{f(r)}+\frac{r^2}{L^2}(dx^2+dy^2+dz^2) \tag{2}$$

together with a static ansatz for the fields,

$$A_\mu=(\phi(r),0,0,0,0)\,, \qquad \psi=\psi(r). \tag{3}$$

Here, without loss of generality ψ can be taken to be real. First, we look for the solutions in normal phase, $\psi=0$. We find ν is constant and

$$\phi=\mu-\frac{\rho}{r^2}\,. \tag{4}$$

Here, μ and ρ are interpreted as a chemical potential and charge density of the dual theory on the boundary, respectively. We also find

$$f(r) = \frac{r^2}{2\alpha}\left[1-\sqrt{1-\frac{4\alpha}{L^2}\left(1-\frac{r_+^4}{r^4}\right)+2\kappa^2\frac{4\alpha\rho^2}{3r^4r_+^2}\left(1-\frac{r_+^2}{r^2}\right)}\right], \tag{5}$$

where we chose the minus sign of the solutions so that we have a solution in the Einstein limit ($\alpha \to 0$). The horizon radius r_+ is defined through the requirement that $f(r_+) = 0$, so we set a constant of integration by imposing this condition at the horizon. We see the above solution becomes the Reissner-Nordström-AdS solution in the Einstein limit. In order to avoid a naked singularity, we need to restrict the parameter range as $\alpha \le L^2/4$. For general α, the solution (5) behaves as

$$f(r) \sim \frac{r^2}{2\alpha}\left[1-\sqrt{1-\frac{4\alpha}{L^2}}\,\right], \tag{6}$$

in the asymptotic region. Hence, we define the effective asymptotic AdS scale by

$$L_{\rm e}^2 = \frac{2\alpha}{1-\sqrt{1-\frac{4\alpha}{L^2}}} \to \begin{cases} L^2\,, & \text{for } \alpha\to 0 \\ \frac{L^2}{2}\,, & \text{for } \alpha \to \frac{L^2}{4} \end{cases}. \tag{7}$$

Here, the maximum value of α is called the Chern-Simons limit. The Hawking temperature is given by

$$T_H = \frac{1}{4\pi} f'(r)e^{\nu(r)}\Big|_{r=r_+}, \tag{8}$$

where a prime denotes derivative with respect to r. This will be interpreted as the temperature of the holographic superconductors.

III. GAUSS-BONNET HOLOGRAPHIC SUPERCONDUCTORS

In order to obtain the solutions in superconducting phase, $\psi \neq 0$, we need to take into account the boundary conditions at the horizon and the AdS boundary. The position of the horizon, r_+, is defined through $f(r_+) = 0$. We can set $\nu(r_+) = 0$ there by rescaling the time coordinate. Then the Einstein equations give:

$$\nu'(r_+) = \frac{2\kappa^2}{3} r_+\left(\psi'(r_+)^2 + \frac{q^2\phi'(r_+)^2\psi(r_+)^2}{f'(r_+)^2e^{2\nu(r_+)}}\right), \tag{9}$$

$$f'(r_+) = \frac{4}{L^2}r_+ - \frac{2\kappa^2}{3}r_+\left(\frac{\phi'(r_+)^2}{2e^{2\nu(r_+)}} + m^2\psi(r_+)^2\right). \tag{10}$$

• Regularity at the horizon for ϕ and ψ gives two conditions:

$$\phi(r_+) = 0, \qquad \psi'(r_+) = \frac{m^2}{f'(r_+)}\psi(r_+)\,. \tag{11}$$

As we want the spacetime to be asymptotically AdS, we look for a solution with

$$\nu(r) = \text{const.}, \qquad f(r) = \frac{r^2}{L_e^2}, \tag{12}$$

at the AdS boundary.

• Asymptotically ($r \to \infty$) the solutions of ϕ and ψ are found to be:

$$\phi(r) \sim \mu - \frac{\rho}{r^2}, \qquad \psi \sim \frac{C_-}{r^{\Delta_-}} + \frac{C_+}{r^{\Delta_+}}, \tag{13}$$

where $\Delta_\pm = 2 \pm \sqrt{4+m^2L_e^2}$. Note that these are not entirely free parameters, as there is a scaling degree of freedom in the equations of motion. As in [4], we impose that ρ is fixed, which determines the scale of the system. For ψ, in order to have a normalizable solution we take $C_- = 0$.

According to the Gauge/Gravity duality [1–3], we can interpret $\langle \mathcal{O}_{\Delta_+} \rangle \equiv C_+$, where $\mathcal{O}_{\Delta_+}$ is the operator with the conformal dimension Δ_+ dual to the scalar field. Thus, we are going to calculate the condensate $\langle \mathcal{O}_{\Delta_+} \rangle$ for fixed charge density. We note that since C_- is regarded as the source term of the operator, we put it zero at the end by using the Gauge/Gravity dictionary [2, 3], which is consistent with taking $C_- = 0$.

Let us change the coordinate and set $z = r_+/r$. Under this transformation, the Einstein, Maxwell and the scalar equations become

$$\left(1 - 2\alpha \frac{z^2}{r_+^2} f\right) \nu' = -\frac{2\kappa^2}{3} \frac{r_+^2}{z^3} \left(\frac{q^2\phi^2\psi^2}{f^2 e^{2\nu}} + \frac{z^4}{r_+^2} \psi'^2 \right) , \tag{14}$$

$$\left(1 - 2\alpha \frac{z^2}{r_+^2} f\right) f' - \frac{2}{z} f + \frac{4r_+^2}{L^2 z^3} = \frac{2\kappa^2}{3} \frac{r_+^2}{z^3} \left[\frac{z^4}{2r_+^2 e^{2\nu}} \phi'^2 + m^2\psi^2 + f \left(\frac{q^2\phi^2\psi^2}{f^2 e^{2\nu}} + \frac{z^4}{r_+^2} \psi'^2 \right) \right] , \tag{15}$$

$$\phi'' - \left(\frac{1}{z} + \nu' \right) \phi' - \frac{2r_+^2}{z^4} \frac{\psi^2}{f} \phi = 0 \, , \tag{16}$$

$$\psi'' - \left(\frac{1}{z} - \nu' - \frac{f'}{f} \right) \psi' + \frac{r_+^2}{z^4} \left(\frac{\phi^2}{f^2 e^{2\nu}} - \frac{m^2}{f} \right) \psi = 0 \, , \tag{17}$$

where the prime now denotes a derivative with respect to z. The region $r_+ < r < \infty$ now corresponds to $0 < z < 1$. If one sets $\tilde{\phi} = \phi/q$, $\tilde{\psi} = \psi/q$ in the action (1), the Maxwell and the scalar equations remain unchanged, while the gravitational coupling of the Einstein equations changes $\kappa^2 \to \kappa^2/q^2$. If one takes the limit $q \to \infty$, the matter sources drop out of the Einstein equations and this is the probe limit. To go beyond the probe limit, we can take either finite q with setting $2\kappa^2 = 1$, or finite $2\kappa^2$ with setting $q = 1$. The paper [5] took the former choice to consider the effects of backreaction of the spacetime metric, but we will take the latter choice in the following.

In order to solve these equations, we focus on near the critical point as in [8, 13], around which the stability was confirmed [14]. It is convenient to introduce a scalar operator as an expansion parameter:

$$\epsilon \equiv \langle \mathcal{O}_{\Delta_+} \rangle \, . \tag{18}$$

As ψ is small near the critical point, we expand ψ from the first order. From Eq. (16), ϕ and ψ are expanded by ϵ^2 subsequently as follows:

$$\phi = \phi_0 + \epsilon^2 \phi_2 + \epsilon^4 \phi_4 + \cdots \, , \tag{19}$$

$$\psi = \epsilon \psi_1 + \epsilon^3 \psi_3 + \epsilon^5 \psi_5 + \cdots \, , \tag{20}$$

and in this situation where starting from the normal phase, the background can be expanded around the Reissner-Nordstöm-AdS spacetime:

$$f = f_0 + \epsilon^2 f_2 + \epsilon^4 f_4 + \cdots \, , \tag{21}$$

$$\nu = \epsilon^2 \nu_2 + \epsilon^4 \nu_4 \cdots \, , \tag{22}$$

where $\epsilon \ll 1$.

While μ is also expanded by the order parameter as

$$\mu = \mu_0 + \epsilon^2 \delta\mu_2 \, , \tag{23}$$

where $\delta\mu_2$ is also positive. So, we find the order parameter as a function of the chemical potential, which is expressed by

$$\epsilon = \langle \mathcal{O}_{\Delta_+} \rangle = \left(\frac{\mu - \mu_0}{\delta\mu_2} \right)^{1/2} \, . \tag{24}$$

The exponent 1/2 is consistent with the Ginzburg-Landau mean field theory for phase transitions. When $\mu = \mu_0$, the order parameter becomes zero, which means the critical value of μ is defined by μ_0 such as $\mu_c = \mu_0$. Now we begin to solve equations order by order.

At zeroth order, we impose $\phi_0(1) = 0$ at the horizon, $z = 1$, and $\phi_0(0) = \mu_0 = \text{const.}$ at the boundary, $z = 0$. Then the solution of Eq. (16) is given by

$$\phi_0(z) = \mu_0(1 - z^2) \, . \tag{25}$$

This gives a relation $\rho = \mu_0 r_+^2$ by the coordinate transformation. Inserting this solution into Eq. (15), we obtain

$$f_0(z) = \frac{r_+^2}{2\alpha}\frac{1}{z^2}\left[1 - \sqrt{1 - \frac{4\alpha}{L^2}(1-z^4) + 2\kappa^2 \frac{4\alpha\mu_0^2}{3r_+^2} z^4(1-z^2)}\,\right], \tag{26}$$

where we chose the minus sign of the solutions so that we have a solution in the Einstein limit. We also used $f_0(1) = 0$ at the horizon as we did in Eq. (5). We find the above solution becomes the Reissner-Nordström-AdS solution in the Einstein limit.

Even at first order, it is difficult to deal with both of the backreaction on the spacetime metric and the Gauss-Bonnet term. Here we will appeal to the matching method used in [6]. Before proceeding to it, let us check the regularity condition at the horizon at this order:

$$\psi_1'(1) = \frac{r_+^2 m^2}{f_0'(1)}\psi_1(1)\ . \tag{27}$$

The behavior of ψ at the boundary is given by

$$\psi_1 \sim D_- z^{\Delta_-} + D_+ z^{\Delta_+}\,, \tag{28}$$

where $\Delta_\pm$ is the same as in Eq. (13). For the boundary conditions, we take D_- to be zero as we did in Eq. (13).

Now we find the solution of ψ_1 under these conditions by the matching method and derive the critical temperature. We can expand ψ_1 in a Taylor series near the horizon as:

$$\psi_1(z) = \psi_1(1) - \psi_1'(1)(1-z) + \frac{1}{2}\psi_1''(1)(1-z)^2 + \cdots \tag{29}$$

Here, we have Eq. (27) for the first order coefficients of ψ_1, and without loss of generality we can take $\psi_1(1) > 0$ to have $\psi_1(z)$ positive. The second order coefficients of ψ_1 are computed by using Eq. (17). Then we find the second derivative at the horizon is expressed by

$$\psi_1''(1) = \frac{1}{2}\left(-3 - \frac{f_0''(1)}{f_0'(1)} + \frac{r_+^2 m^2}{f_0'(1)}\right)\psi_1'(1) - \frac{r_+^2 \phi_0'(1)^2}{2 f_0'(1)^2}\psi_1(1)\,. \tag{30}$$

After eliminating $\psi_1'(1)$ from above equation by using Eq. (27), an approximate solution near the horizon is given by

$$\psi_1(z) = \psi_1(1) - \frac{r_+^2 m^2}{f_0'(1)}\psi_1(1)(1-z) + \left[-\frac{r_+^2 m^2}{4 f_0'(1)}\left(3 + \frac{f_0''(1)}{f_0'(1)} - \frac{r_+^2 m^2}{f_0'(1)}\right) - \frac{r_+^2}{4}\frac{\phi_0'(1)^2}{f_0'(1)^2}\right]\psi_1(1)(1-z)^2 + \cdots \tag{31}$$

Here, $\psi_1(1)$ is still unknown.

On the other hand, from (28), ψ_1 in the asymptotic region are given by

$$\psi_1(z) \sim D_+ z^{\Delta_+}\,. \tag{32}$$

We have set $D_- = 0$ from the boundary condition. Here D_+ is another unknown constant.

Now we try to connect the solutions Eq. (31) and (32) smoothly at z_m in order to obtain $\psi_1(1)$ and D_+. As we shall see below the choice of z_m does not change the qualitative features of solutions. In order to connect those solutions smoothly, we require the following two conditions:

$$z_m^{\Delta_+} D_+ = \psi_1(1) - \frac{r_+^2 m^2}{f_0'(1)}(1-z_m)\psi_1(1) + \left[-\frac{r_+^2 m^2}{4 f_0'(1)}\left(3 + \frac{f_0''(1)}{f_0'(1)} - \frac{r_+^2 m^2}{f_0'(1)}\right) - \frac{r_+^2}{4}\frac{\phi_0'(1)^2}{f_0'(1)^2}\right](1-z_m)^2\psi_1(1)\,, \tag{33}$$

$$\Delta_+ z_m^{\Delta_+ - 1} D_+ = \frac{r_+^2 m^2}{f_0'(1)}\psi_1(1) - 2\left[-\frac{r_+^2 m^2}{4 f_0'(1)}\left(3 + \frac{f_0''(1)}{f_0'(1)} - \frac{r_+^2 m^2}{f_0'(1)}\right) - \frac{r_+^2}{4}\frac{\phi_0'(1)^2}{f_0'(1)^2}\right](1-z_m)\psi_1(1)\,. \tag{34}$$

From Eqs (33) and (34), we find the relation between $\psi_1(1)$ and D_+,

$$D_+ = \frac{2z_m}{2z_m + (1-z_m)\Delta_+} z_m^{-\Delta_+}\left(1 - \frac{1-z_m}{2}\frac{r_+^2 m^2}{f_0'(1)}\right)\psi_1(1)\,. \tag{35}$$

Plugging this back into Eq. (34), we find the following relation in order to get a non-trivial solution, $\psi_1(1) \neq 0$,

$$\begin{aligned} &\frac{2\Delta_+}{2z_m + (1-z_m)\Delta_+} - \left(\frac{(1-z_m)\Delta_+}{2z_m + (1-z_m)\Delta_+} + \frac{5 - 3z_m}{2}\right)\frac{r_+^2 m^2}{f_0'(1)} \\ &\qquad - \frac{(1-z_m) r_+^2 m^2}{2}\frac{f_0''(1)}{f_0'(1)^2} + \frac{1-z_m}{2}\frac{r_+^4 m^4}{f_0'(1)^2} - \frac{(1-z_m) r_+^2}{2}\frac{\phi_0'(1)^2}{f_0'(1)^2} = 0\,. \end{aligned} \tag{36}$$

Similarly, plugging Eq. (35) back in Eq. (33) we can get the solution of $\psi_1(1)$ and then D_+ as well in principle. But what we want to know here is the critical temperature rather than deriving the solution of ψ up to this order, so we are going to focus on it in the following. For this purpose, putting the values of $f_0'(1)$, $f_0''(1)$ and $\phi_0'(1)$ in the above relation, then Eq. (36) yields the equation for μ_0:

$$\begin{aligned}&4\kappa^4\frac{L^4}{36r_+^4}\left[\frac{2\Delta_+}{2z_m+(1-z_m)\Delta_+}-(1-z_m)m^2\alpha\right]\mu_0^4\\&-\frac{(1-z_m)L^4}{8r_+^2}\left[1+2\kappa^2\left\{\left(\frac{16}{3(1-z_m)L^2}+\frac{m^2}{3}\right)\frac{\Delta_+}{2z_m+(1-z_m)\Delta_+}+\left(\frac{5-3z_m}{6(1-z_m)}+\frac{5}{6}\right)m^2-\frac{8m^2}{3L^2}\alpha\right\}\right]\mu_0^2\\&+\frac{(1-z_m)\Delta_+}{2z_m+(1-z_m)\Delta_+}\left(\frac{2}{1-z_m}+\frac{m^2L^2}{4}\right)+\frac{2-z_m}{4}m^2L^2+\frac{1-z_m}{32}m^4L^4-(1-z_m)m^2\alpha=0\,.\end{aligned}\tag{37}$$

In order to solve the above equation with respect to μ_0, we assume $\kappa^2 \ll 1$ in the following. This means that all functions are expanded by κ^2 as well. That is, solutions near the phase transition are obtained in the small backreaction approximation together with the matching method. As the first term disappears from the above equation, we can solve it easily and we get

$$\begin{aligned}\mu_0 &= \sqrt{\frac{8}{1-z_m}}\frac{r_+}{L^2}\left[\frac{(1-z_m)\Delta_+}{2z_m+(1-z_m)\Delta_+}\left(\frac{2}{1-z_m}+\frac{m^2L^2}{4}\right)+\frac{2-z_m}{4}m^2L^2+\frac{1-z_m}{32}m^4L^4-(1-z_m)m^2\alpha\right]^{1/2}\\&\times\left[1-2\kappa^2\left\{\left(\frac{16}{3(1-z_m)L^2}+\frac{m^2}{3}\right)\frac{\Delta_+}{4z_m+2(1-z_m)\Delta_+}+\left(\frac{5-3z_m}{6(1-z_m)}+\frac{5}{6}\right)\frac{m^2}{2}-\frac{4m^2}{3L^2}\alpha\right\}\right],\end{aligned}\tag{38}$$

where μ_0 is positive. Combining Eq. (38) with the relation $\mu_0 = \rho/r_+^2$, which is given under Eq. (25), we find r_+ is given by

$$\begin{aligned}r_+ &= \frac{\rho^{1/3}}{\pi L^{4/3}}\left(\frac{1-z_m}{8}\right)^{1/6}\left[\frac{8\Delta_++(1-z_m)\Delta_+m^2L^2}{8z_m+4(1-z_m)\Delta_+}+\frac{2-z_m}{4}m^2L^2+\frac{1-z_m}{32}m^4L^4-(1-z_m)m^2\alpha\right]^{-1/6}\\&\times\left[1+\frac{2\kappa^2}{L^2}\left\{\frac{\Delta_+}{12z_m+6(1-z_m)\Delta_+}\left(\frac{m^2L^2}{3}+\frac{16}{3(1-z_m)}\right)+\frac{5-4z_m}{1-z_m}\frac{m^2L^2}{18}-\frac{4m^2}{9}\alpha\right\}\right].\end{aligned}\tag{39}$$

The Hawking temperature Eq. (8) up to this order becomes

$$T_H = -\frac{f_0'(1)e^{\nu_0(1)}}{4\pi r_+} = \frac{r_+}{\pi L^2}\left(1-2\kappa^2\frac{L^2}{6r_+^2}\mu_0^2\right).\tag{40}$$

The critical temperature is defined at the point where the order parameter becomes zero, which leads to $T_H = T_c$ at $\mu_0 = \mu_c$. Eliminating $\mu_0(=\mu_c)$ from the above temperature T_H using Eq. (38), we get the critical temperature

$$T_c = \frac{r_+}{\pi L^2}\left[1-\frac{2\kappa^2}{L^2}\left\{\frac{4\Delta_+}{6z_m+3(1-z_m)\Delta_+}\left(\frac{2}{1-z_m}+\frac{m^2L^2}{4}\right)+\frac{2-z_m}{3(1-z_m)}m^2L^2+\frac{m^4L^4}{24}-\frac{4m^2\alpha}{3}\right\}\right].\tag{41}$$

Substituting r_+ with Eq. (39), we obtain the critical temperature of the form

$$T_c = T_1\left(1-\frac{2\kappa^2}{L^2}T_2\right),\tag{42}$$

where

$$T_1 = \frac{\rho^{1/3}}{\pi L^{4/3}}\left(\frac{1-z_m}{8}\right)^{1/6}\left[\frac{8\Delta_++(1-z_m)\Delta_+m^2L^2}{8z_m+4(1-z_m)\Delta_+}+\frac{2-z_m}{4}m^2L^2+\frac{1-z_m}{32}m^4L^4-(1-z_m)m^2\alpha\right]^{-1/6}\tag{43}$$

$$T_2 = \frac{2\Delta_+}{2z_m+(1-z_m)\Delta_+}\left(\frac{5m^2L^2}{36}+\frac{8}{9(1-z_m)}\right)+\frac{7-2z_m}{1-z_m}\frac{m^2L^2}{18}+\frac{m^4L^4}{24}-\frac{8m^2\alpha}{9}.\tag{44}$$

IV. CONCLUSION

We presented an analytic treatment near the phase transition for the critical temperature of $(3+1)$-dimensional holographic superconductors in Einstein-Gauss-Bonnet gravity with backreaction by matching the solution expanded

from infinity with that expanded from the horizon. In this method, there is an ambiguity in the choice of the matching radius. However, the result turns out to be fairly insensitive to the choice of it. The result reproduces our previous analytic calculation for $m^2 = -3/L^2$, $\kappa^2 = 0$ with the choice $z_m = 1/2$, which was in good agreement with the numerical result [6]. It also agrees with [8] for $m^2 = -4/L^2$, $\kappa^2 = \alpha = 0$ with the choice $z_m = 1/2$. It may be noted that the choice $z_m \sim 0.5$ is roughly equal to $z = \sqrt{r_+/L}$, corresponding to the geometrical mean of the horizon radius and the AdS scale, $r = \sqrt{r_+ L}$. We found that the coefficient of the corrections due to backreaction, T_2, is positive definite irrespective of the value of α. This means the effects of the backreaction makes condensation harder. This result also agrees with the numerical results obtained in [15–17].

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